

# Large Mixing Angle MSW Solution in $S_3$ Flavor Symmetry

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## ABSTRACT

We have investigated phenomenological implications on the neutrino flavor mixings in the  $S_{3L} \times S_{3R}$  symmetric mass matrices including symmetry breaking terms. We have shown how to get the large mixing angle MSW solution,  $\sin^2 2\theta_\odot = 0.65 \sim 0.97$  and  $\Delta m_\odot^2 = 10^{-5} \sim 10^{-4} \text{eV}^2$ , in this model. It is found that the structure of the lepton mass matrix in our model is stable against radiative corrections although the model leads to nearly degenerate neutrinos.

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Recent Super-Kamiokande data of atmospheric neutrinos [1] have provided a more solid evidence of the neutrino oscillation, which corresponds to the nearly maximal neutrino flavor mixing. The observed solar neutrino deficit is also an indication of a different sort of the neutrino oscillation [2]. For the solar neutrino problem, four solutions are still allowed. Those are large mixing angle (LMA) MSW, small mixing angle (SMA) MSW [3], vacuum oscillation (VO) and low  $\Delta m^2$  (LOW) solutions [4].

Those data give constraints on the structure of the lepton mass matrices in the three family model [5, 7, 8], which may suggest the some flavor symmetry [6, 9]. There is a typical texture of the lepton mass matrix with the nearly maximal mixing of flavors, which is derived from the symmetry of the lepton flavor democracy [7], or from the  $S_{3L} \times S_{3R}$  symmetry of the left-handed Majorana neutrino mass matrix [8]. This texture have given a prediction for the neutrino mixing  $\sin^2 2\theta_{\text{atm}} = 8/9$ . The mixing for the solar neutrino depends on the symmetry breaking pattern of the flavor such as  $\sin^2 2\theta_{\odot} = 1$  or  $\ll 1$ . However, the LMA-MSW solution,  $\sin^2 2\theta_{\odot} = 0.65 \sim 0.97$  and  $\Delta m_{\odot}^2 = 10^{-5} \sim 10^{-4} \text{eV}^2$ , has not been obtained in the previous works [7, 8].

In this paper, we study how to get the LMA-MSW solution in the  $S_{3L} \times S_{3R}$  symmetric mass matrices including symmetry breaking terms. Furthermore, we discuss the stability of the neutrino mass matrix against radiative corrections since the model predicts nearly degenerate neutrinos.

We assume that oscillations need only account for the solar and atmospheric neutrino data. Since the result of LSND [10] awaits confirmation by KARMEN experiment [11], we do not take into consideration the LSND data in this paper. Our starting point as to the neutrino mixing is the large  $\nu_{\mu} \rightarrow \nu_{\tau}$  oscillation of atmospheric neutrinos with  $\Delta m_{\text{atm}}^2 = (2 \sim 6) \times 10^{-3} \text{eV}^2$  and  $\sin^2 2\theta_{\text{atm}} \geq 0.84$ , which is derived from the recent data of the atmospheric neutrino deficit at Super-Kamiokande [1]. The mass difference scales of the solar neutrinos are  $\Delta m_{\odot}^2 = 10^{-10} \sim 10^{-4} \text{eV}^2$  depending on the four solutions [4].

The texture of the charged lepton mass matrix was presented based on the  $S_{3L} \times S_{3R}$  symmetry as follows [7, 8, 12]:

$$M_\ell = \frac{c_\ell}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + M_\ell^{(c)}, \quad (1)$$

where the second matrix is the flavor symmetry breaking one. The unitary matrix  $V_\ell$ , which diagonalizes the mass matrix  $M_\ell$ , is given as  $V_\ell = FL$ , where

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \quad (2)$$

diagonalizes the democratic matrix and  $L$  depends on the mass correction term  $M_\ell^{(c)}$ .

Let us turn to the neutrino sector. The neutrino mass matrix is different from the democratic one if they are Majorana particles. The  $S_{3L}$  symmetric mass term is given as follows:

$$c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + c_\nu r \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (3)$$

where  $c_\nu$  and  $r$  are arbitrary parameters. The eigenvalues of this matrix are easily obtained by using the orthogonal matrix  $F$  in eq.(2) as  $c_\nu(1, 1, 1 + 3r)$ , which means that there are at least two degenerate masses in the  $S_{3L}$  symmetric Majorana mass matrix [8, 13, 14].

The simplest breaking terms of the  $S_{3L}$  symmetry are added in (3,3) and (2,2) entries. Therefore, the neutrino mass matrix is written as

$$M_\nu = c_\nu \begin{pmatrix} 1+r & r & r \\ r & 1+r+\epsilon & r \\ r & r & 1+r+\delta \end{pmatrix}, \quad (4)$$

in terms of small breaking parameters  $\epsilon$  and  $\delta$ . In order to explain both solar and atmospheric neutrinos in this mass matrix,  $r \ll 1$  should be satisfied. In other words, three neutrinos should be nearly degenerate.<sup>2</sup> However, there is no reason why  $r$  is very

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<sup>2</sup> $r = -2/3$  also gives nearly degenerate neutrinos [13]. However, solar and atmospheric neutrinos are not explained by simple breaking terms in eq.(4).

small in this framework. In order to answer this question, we need a higher symmetry of flavors such as the  $O_{3L} \times O_{3R}$  model [9]. We do not address this problem in this paper.

We start with discussing the simple case of  $\epsilon = 0$  and  $\delta \gg r$ , in which the  $S_{2L}$  symmetry is preserved but the  $S_{3L}$  symmetry is broken. Mass eigenvalues are given as

$$m_1 = 1, \quad m_2 \simeq 1 + 2r, \quad m_3 \simeq 1 + r + \delta, \quad (5)$$

in the  $c_\nu$  unit. We easily obtain  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 \simeq 2c_\nu^2 \delta$  and  $\Delta m_\odot^2 = \Delta m_{21}^2 \simeq 4c_\nu^2 r$ . The neutrino mass matrix is diagonalized by the orthogonal matrix  $U_\nu$  such as  $U_\nu^T M_\nu U_\nu$ , where

$$U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{r}{\delta} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{r}{\delta} \\ 0 & -\sqrt{2}\frac{r}{\delta} & 1 \end{pmatrix}, \quad (6)$$

in which the first and second family mixes maximally due to the  $S_{2L}$  symmetry. This maximal mixing is completely canceled out by the charged lepton sector in the neutrino mixing matrix (MNS mixing matrix)  $U_{\alpha i}$  [15], which is determined by the product of  $V_\ell^\dagger$  and  $U_\nu$  as follows:

$$U = V_\ell^\dagger U_\nu = L^\dagger F^T U_\nu \simeq \begin{pmatrix} 1 & \frac{1}{\sqrt{3}}L_{21} & -\sqrt{\frac{2}{3}}L_{21} \\ L_{12} & \frac{1}{\sqrt{3}}(1 + 2\frac{r}{\delta} + \sqrt{2}L_{32}) & -\sqrt{\frac{2}{3}}(1 - \frac{r}{\delta} + \frac{1}{\sqrt{2}}L_{32}) \\ L_{13} & \sqrt{\frac{2}{3}}(1 - \frac{r}{\delta} + \frac{1}{\sqrt{2}}L_{23}) & \frac{1}{\sqrt{3}}(1 + 2\frac{r}{\delta} - \sqrt{2}L_{23}) \end{pmatrix}, \quad (7)$$

where  $L_{ij}$  are components of the correction matrix  $L$  in the charged lepton sector. We take  $L_{ii} \simeq 1$  ( $i = 1, 2, 3$ ) and  $L_{31} \ll L_{21} \ll 1$  like mixings in the quark sector. The CP violating phase is also neglected. This case corresponds to the SMA-MSW solution of the solar neutrino. In this MNS mixing matrix, we have:

$$U_{e3} \simeq -\sqrt{2}U_{e2}, \quad (8)$$

which means that  $U_{e3}$  is predicted if the solar neutrino data will be confirmed in the future. The long baseline (LBL) experiments provide an important test of the model

since the oscillation of  $\nu_\mu \rightarrow \nu_e$  is predicted as follows:

$$P(\nu_\mu \rightarrow \nu_e) \simeq \frac{4}{3} \sin^2 2\theta_\odot \sin^2 \frac{\Delta m_{31}^2 L}{4E} . \quad (9)$$

Putting  $\sin^2 2\theta_\odot$  of the SMA-MSW solution [4], we obtain  $P(\nu_\mu \rightarrow \nu_e) = 10^{-3} \sim 10^{-2}$  in the relevant LBL experiment. These results with the SMA-MSW solution of the solar neutrino are maintained as far as  $\epsilon \ll r$ .

Let us consider the case of  $\epsilon \neq 0$  with  $\delta \gg \epsilon \simeq r$ , in which  $S_{3L}$  symmetry is completely broken. Then neutrino mass eigenvalues are given as

$$m_1 \simeq 1 + \frac{1}{2}\epsilon + r - \frac{1}{2}\sqrt{\epsilon^2 + 4r^2}, \quad m_2 \simeq 1 + \frac{1}{2}\epsilon + r + \frac{1}{2}\sqrt{\epsilon^2 + 4r^2}, \quad m_3 \simeq 1 + r + \delta, \quad (10)$$

in the  $c_\nu$  unit. Then we have

$$\Delta m_{32}^2 \simeq 2c_\nu^2 \delta, \quad \Delta m_{21}^2 \simeq 2c_\nu^2 \sqrt{\epsilon^2 + 4r^2}. \quad (11)$$

The orthogonal matrix  $U_\nu$  is given as

$$U_\nu \simeq \begin{pmatrix} t & \sqrt{1-t^2} & \frac{r}{\delta} \\ -\sqrt{1-t^2} & t & \frac{r}{\delta-\epsilon} \\ \frac{r}{\delta}(\sqrt{1-t^2}-t) & -\frac{r}{\delta-\epsilon}(t+\sqrt{1-t^2}) & 1 \end{pmatrix}, \quad (12)$$

where

$$t^2 = \frac{1}{2} + \frac{1}{2} \frac{\epsilon}{\sqrt{\epsilon^2 + 4r^2}}. \quad (13)$$

In order to find the structure of the MNS matrix  $U_{\alpha i}$ , we show  $F^T U_\nu$  as follows:

$$F^T U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(t + \sqrt{1-t^2}) & \frac{1}{\sqrt{2}}(\sqrt{1-t^2} - t) & -\frac{1}{\sqrt{2}} \frac{\epsilon r}{\delta(\delta-\epsilon)} \\ \frac{1}{\sqrt{6}}(t - \sqrt{1-t^2})(1 + \frac{2r}{\delta}) & \frac{1}{\sqrt{6}}(t + \sqrt{1-t^2})(1 + \frac{2r}{\delta-\epsilon}) & -\frac{2}{\sqrt{6}}(1 - \frac{r}{\delta}) \\ \frac{1}{\sqrt{3}}(t - \sqrt{1-t^2})(1 - \frac{r}{\delta}) & \frac{1}{\sqrt{3}}(t + \sqrt{1-t^2})(1 - \frac{r}{\delta-\epsilon}) & \frac{1}{\sqrt{3}}(1 + \frac{2r}{\delta}) \end{pmatrix}. \quad (14)$$

The mixing angle between the first and second flavor depends on  $t$ , which is determined by  $r/\epsilon$ . It becomes the maximal angle in the case of  $t = 1$  ( $r/\epsilon = 0$ ) and the minimal one in the case of  $t = 1/\sqrt{2}$  ( $\epsilon/r = 0$ ). It is emphasized that the relevant value of  $r/\epsilon$  leads easily to  $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ , which corresponds to the LMA-MSW solution. The case of  $t = 1/\sqrt{2}$  may correspond rather to the VO solution.

In order to get the MNS mixing matrix  $U_{\alpha i}$ , the correction matrix  $L^\dagger$  in the charged lepton sector should be multiplied such as  $L^\dagger F^T U_\nu$ . Then we obtain:

$$\begin{aligned}
U_{e1} &\simeq \frac{1}{\sqrt{2}}(t + \sqrt{1-t^2}) + \frac{1}{\sqrt{6}}(t - \sqrt{1-t^2})L_{21} , \\
U_{e2} &\simeq \frac{1}{\sqrt{2}}(\sqrt{1-t^2} - t) + \frac{1}{\sqrt{6}}(t + \sqrt{1-t^2})L_{21} , \\
U_{e3} &\simeq -\frac{2}{\sqrt{6}}(1 - \frac{r}{\delta})L_{21} , \\
U_{\mu 1} &\simeq \frac{1}{\sqrt{6}}(t - \sqrt{1-t^2})(1 + \frac{2r}{\delta}) + \frac{1}{\sqrt{2}}(t + \sqrt{1-t^2})L_{12} , \\
U_{\mu 2} &\simeq \frac{1}{\sqrt{6}}(t + \sqrt{1-t^2})(1 + \frac{2r}{\delta}) + \frac{1}{\sqrt{2}}(\sqrt{1-t^2} - t)L_{12} , \\
U_{\mu 3} &\simeq -\frac{1}{\sqrt{6}}(2 - \frac{2r}{\delta} - \sqrt{2}L_{32}) , \\
U_{\tau 1} &\simeq \frac{1}{\sqrt{3}}(t - \sqrt{1-t^2})(1 - \frac{r}{\delta} + \frac{1}{\sqrt{2}}L_{23}) , \\
U_{\tau 2} &\simeq \frac{1}{\sqrt{3}}(t + \sqrt{1-t^2})(1 - \frac{r}{\delta} + \frac{1}{\sqrt{2}}L_{23}) , \\
U_{\tau 3} &\simeq \frac{1}{\sqrt{3}}(1 + \frac{2r}{\delta} - \sqrt{2}L_{23}) ,
\end{aligned} \tag{15}$$

where  $L_{ii} \simeq 1 (i = 1, 2, 3)$  are taken and  $L_{31}, L_{13}$  are neglected. The CP violating phase is also neglected.  $U_{e3}$  depends on  $L_{21}$ , which is determined by  $M_\ell^{(c)}$  in eq.(1). The MNS mixings in eqs.(15) agree with the numerical one (without any approximations) within a few percent error.

We should carefully discuss the stability of our results against radiative corrections since the model predicts nearly degenerate neutrinos. When the texture of the mass matrix is given at the  $S_{3L} \times S_{3R}$  symmetry energy scale, radiative corrections are not negligible at the electroweak (EW) scale. The runnings of the neutrino masses and mixings have been studied by using the renormalization group equations (RGE's) [16, 17, 18].

Let us consider the basis, in which the mass matrix of the charged leptons is diagonal. The neutrino mass matrix in eq.(4) is transformed into  $V_\ell^\dagger M_\nu V_\ell$ . Taking  $V_\ell \simeq F$

because of  $L$  being close to the unit matrix, we obtain the mass matrix at the high energy scale:

$$F^T M_\nu F = \overline{M}_\nu = c_\nu \begin{pmatrix} 1 + \frac{\epsilon}{2} & -\frac{\epsilon}{2\sqrt{3}} & -\frac{1}{\sqrt{6}}\epsilon \\ -\frac{\epsilon}{2\sqrt{3}} & 1 + \frac{1}{6}\epsilon + \frac{2}{3}\delta & \frac{\sqrt{2}}{6}\epsilon - \frac{\sqrt{2}}{3}\delta \\ -\frac{1}{\sqrt{6}}\epsilon & \frac{\sqrt{2}}{6}\epsilon - \frac{\sqrt{2}}{3}\delta & 1 + \frac{1}{3}\epsilon + \frac{1}{3}\delta + 3r \end{pmatrix}. \quad (16)$$

The radiatively corrected mass matrix in the MSSM at the EW scale is given as  $R_G \overline{M}_\nu R_G$ , where  $R_G$  is given by RGE's [18] as

$$R_G \simeq \begin{pmatrix} 1 + \eta_e & 0 & 0 \\ 0 & 1 + \eta_\mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (17)$$

where  $\eta_e$  and  $\eta_\mu$  are

$$\eta_i = 1 - \sqrt{\frac{I_i}{I_\tau}} \quad (i = e, \mu), \quad (18)$$

with

$$I_i \equiv \exp \left( \frac{1}{8\pi^2} \int_{\ln(M_Z)}^{\ln(M_R)} y_i^2 dt \right). \quad (19)$$

Here  $y_i$  ( $i = e, \mu$ ) are Yukawa couplings and the  $M_R$  scale is taken as the  $S_{3L} \times S_{3R}$  symmetry energy scale. We transform back this neutrino mass matrix  $R_G \overline{M}_\nu R_G$  into the basis where the charged lepton mass matrix is the democratic one at the EW scale:

$$F R_G \overline{M}_\nu R_G F^T \simeq c_\nu \begin{pmatrix} 1 + \bar{r} & \bar{r} & \bar{r} \\ \bar{r} & 1 + \epsilon + \bar{r} & \bar{r} \\ \bar{r} & \bar{r} & 1 + \delta + \bar{r} \end{pmatrix} + 2\eta_R c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (20)$$

where

$$\bar{r} = r - \frac{2}{3}\eta_R. \quad (21)$$

Here we take  $\eta_R \equiv \eta_e \simeq \eta_\mu$ , which is a good approximation [18]. Its numerical value depends on  $\tan\beta$  as:  $10^{-2}$ ,  $10^{-3}$  and  $10^{-4}$  for  $\tan\beta = 60$ , 10, and 1, respectively. As seen in eq.(4) and eq.(20), radiative corrections are absorbed into the original parameters  $r$ ,  $\epsilon$  and  $\delta$  in the leading order. Thus the structure of the mass matrix is stable against radiative corrections although our model leads to nearly degenerate neutrinos.

Let us present numerical results. We take  $L_{12} = -L_{21} = \sqrt{m_e/m_\mu}$  and  $L_{23} = -L_{32} = -m_\mu/m_\tau$ , which are suggested from the ones in the quark sector, in eqs.(15). We show the result in the case of  $\delta = 0.05$  as a typical case.<sup>3</sup> Putting  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2 = 3 \times 10^{-3}$  in eq.(11), we get  $c_\nu = 0.18\text{eV}$ , which is consistent with the double beta decay experiment [20].<sup>4</sup> Taking  $\epsilon = 0.002$  as a typical value, predictions of  $\sin^2 2\theta_{\text{atm}}$  and  $\sin^2 2\theta_\odot$  are shown versus  $r$  in fig.1. It is found that the predicted solar neutrino mixing lies in the region of the LMA-MSW solution if  $r/\epsilon = 0.1 \sim 0.5$  is taken, while the mixing of the atmospheric neutrino changes slowly. In this parameter region,  $\Delta m_\odot^2 = (1 \sim 2) \times 10^{-4}\text{eV}^2$  is predicted. As far as  $\delta = \lambda^2 \sim \lambda$  and  $\epsilon = \lambda^4 \sim \lambda^3$ , where  $\lambda \simeq 0.22$ , obtained results are similar to the ones in fig.1. Thus the LMA-MSW solution with  $\sin^2 2\theta_{\text{atm}} \geq 0.9$  is easily realized by taking a relevant  $r/\epsilon$  in this model.

We have investigated phenomenological implications on the neutrino flavor mixings in the  $S_{3L} \times S_{3R}$  symmetric mass matrices including symmetry breaking terms. We have shown how to get the LMA-MSW solution in this model. The non-zero value of the symmetric parameter  $r$  is essential in order to get  $\sin^2 2\theta_\odot = 0.65 \sim 0.97$ . However, there is no reason that  $r$  is very small in the  $S_{3L} \times S_{3R}$  symmetry, and so we need its extension, for example, the  $O_{3L} \times O_{3R}$  model [9], which leads to naturally the small  $r$  and the unique prediction of the LMA-MSW solution. It is found that radiative corrections are absorbed into the original parameters  $r$ ,  $\epsilon$  and  $\delta$ . Therefore, the structure of the mass matrix is stable against radiative corrections although it leads to nearly degenerate neutrinos. Furthermore, the neutrino mass matrix can be modified by introducing the CP violating phase [19]. We wait for results in KamLAND experiment [21] as well as new solar neutrino data.

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<sup>3</sup>Parameters  $r$ ,  $\epsilon$  and  $\delta$  are assumed to be real. If they are taken to be complex, the CP violation can be predicted as in ref.[19].

<sup>4</sup>The result is consistent with the constraint of the double beta decay experiment as far as  $\delta \geq 0.04$ .



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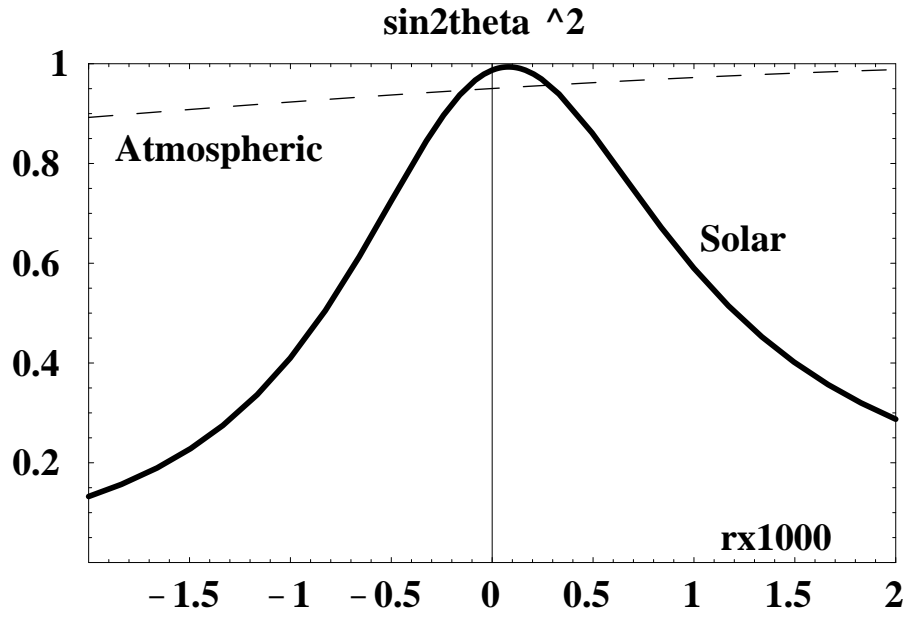


Figure 1:

Fig. 1: The  $r$  dependence of  $\sin^2 2\theta_{\text{atm}}$  and  $\sin^2 2\theta_{\odot}$ .  $c_\nu = 0.18\text{eV}$ ,  $\delta = 0.05$  and  $\epsilon = 0.002$  are taken.